

## Lec 5 dr mervat

Correlation :-

The relation between observation

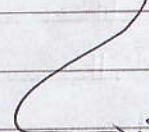
$$\text{Correlation} = \frac{\sigma_{xy}}{\sigma_x \cdot \sigma_y} = \frac{\text{Covariance}}{\text{Var}(x) \cdot \text{Var}(y)}$$

How to get standered deviation

$$1. \text{ mean} = \frac{\sum \text{observation}}{\text{number}}$$

$$2. \text{ Residual} = \text{mean} - \text{observation}$$

$$\sigma_s = \sqrt{\frac{\sum VV}{n-1}}$$


 st. deviation for a single observation

$$\text{st. dev of mean} = \frac{\sigma_s}{\sqrt{n}}$$



## Variance Covariance Matrix

$$\begin{bmatrix} \sigma_x^2 & \text{Cov}_{xy} & \text{Cov}_{xz} \\ \text{Cov}_{yx} & \sigma_y^2 & \text{Cov}_{yz} \\ \text{Cov}_{zx} & \text{Cov}_{zy} & \sigma_z^2 \end{bmatrix}$$

diagonal will not contain any negative sign

Correlation :-

 $0 < r < .35$  weak $.35 < r < .75$  medium $.75 < r < 1$  strong $r = 1 \rightarrow$  in the Correlation between the Variable and itself

$$r = \frac{\sigma_{xx}}{\sigma_x \sigma_x} = 1$$



Example :- Find The Correlation between the weight and the height

weight	170	155	190	140	145	165	185	180	155	175
height	70	70	70	66	68	72	70	73	60	70

Solution

weight	Residual	$V_w^2$	height	Residual	$V_H^2$	$V_w V_H$
170	-4	16	70	0	0	0
155	11	121	70	0	0	0
190	-24	576	70	4	16	48
140	26	676	66	16	256	104
145	21	441	68	4	16	42
165	1	1	72	-2	4	-2
185	-19	361	70	0	0	0
180	-14	196	73	3	9	42
155	11	121	60	-11	121	-11
175	-9	81	70	0	0	0



$\Sigma 1660$   $\Sigma 2592$   $\Sigma 700$

$\Sigma 245$

$$\begin{aligned} \text{mean of weight } m_w &= \frac{1660}{10} = 166 \\ \sim \sim \text{height } m_h &= \frac{700}{10} = 70 \end{aligned}$$

$$\sigma_w = \sqrt{\frac{\sum V_w^2}{n-1}} = \sqrt{\frac{2592}{9}} = 16.97$$

$$\sigma_H = \sqrt{\frac{\sum V_H^2}{n-1}} = \sqrt{\frac{121}{9}} = 2.05$$

$$\text{Covariance} = \frac{\sum V_w V_H}{n-1} = \frac{245}{9} = 27.22$$

$$\text{Correlation} = \frac{27.22}{16.97(2.05)} = 0.78 \text{ strong}$$

$$\text{st. dev of mean } \sigma_{Hm} = \frac{16.97}{\sqrt{10}} = 5.34$$

$$\sigma_{Vm} = \frac{2.05}{\sqrt{10}} = 0.65$$



last result will be like this

For weight  $\rightarrow 166 \pm 5.34$

For Height  $\rightarrow 70 \pm 0.65$





Correlation is between two dependant variables expressed by coefficient of Correlation  
The Correlation could be Positive or negative

$\rho$  changes from  $+1$  to  $-1$

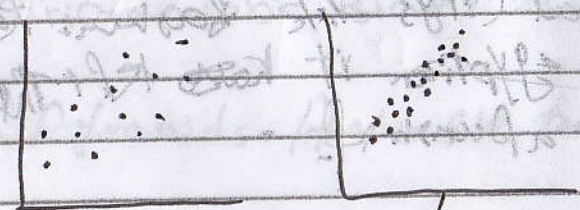
Scatter Plot



Positive

Negative

No Correlation



Weak Correlation

Strong Correlation

Theory of error "lec 5"

1- standard deviation  $\sigma = \sqrt{\frac{\sum v^2}{n-1}}$  Residual

2- Variance  $= \frac{\sum v^2}{n-1}$

↳ we take  $n$  times but not the right Value

3- Covariance  $= \frac{\sum U_x U_y}{n-1}$

4-  $\rho = \frac{\sigma_{U_{xy}}}{\sigma_x \cdot \sigma_y}$

5 Correlation is mostly 1 between the variable and itself

6- Variance Covariance matrix

$$\begin{bmatrix} \sigma_x^2 & \text{Cov} & \text{Cov} \\ - & \sigma_y^2 & - \\ - & - & \sigma_z^2 \end{bmatrix}$$



7 less standard deviation gives you high accuracy

if you get some measurements you could get the mean of them

\* The expected value is the accurate one

How to get st dev for an expected value

• Here you calculate  $e \rightarrow$  expected error

st. deviation  $\sqrt{\frac{[e \cdot e]}{n}}$   $\rightarrow$  we don't need sign  
 $\rightarrow$  The Right Value between those measurements

$x_i$	$\Delta x$	$\Delta x^2$	$v$	$v^2$
$153^\circ 23' 37.6$	0.6	0.36	-0.04	
30.9	-0.1	0.01	0.66	
31.6	0.6		0.861	
31.3	0.3		-0.004	
31.8	0.8		-0.24	
31.8	0.8		-0.24	
31.7	0.7		-0.44	
31.8	0.8		-0.24	
31.6	0		0.50	
32.1	1.1		-0.54	
	5.6	4.44		1.340

and you've received an approximate value for  $x$

$x^0 = 153^\circ 23' 3'' \rightarrow$  it's not better than expected or mean

$\Delta \bar{x} \therefore$  try to make  $\Delta x$  close to mean

$$\Delta \bar{x} = \frac{5.6}{10} = 0.56$$

$n \leftarrow$



\* and know to get Residual - The distance from the mean

$$v = X - \bar{X}$$

\* now we've got  $v^2$  you could get

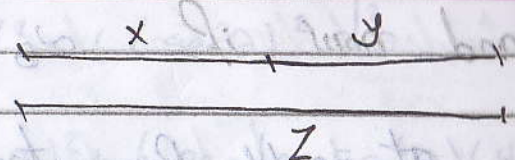
the st. deviation  $\sigma = \sqrt{\frac{\sum v^2}{n-1}}$   
 ↳ of approximation

$$* [v] = [\Delta x^2] = \frac{[\Delta x]^2}{n}$$

\* and now to get st. dev of mean

$$\bar{X} = 153 \text{ } 23' \text{ } 3''$$

① math model if you've



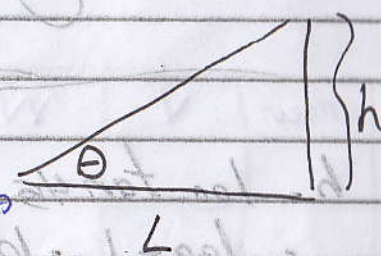
measured  $x \approx y$  and calculated  $\sigma_x \approx \sigma_y$

How to get  $\sigma_Z$   $Z = x + y$

$$\sigma_Z^2 = \sigma_x^2 + \sigma_y^2 \quad \text{"linear math model"}$$

$$h = L \tan \theta$$

you could get  $\sigma_L \approx \sigma_\theta$



but How about  $\sigma_h$  you first have to make linearization

- 1- differentiation
- 2- logarithmic